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Freiburg, 26.10.2011

Übungen zur Experimentalphysik I, WS 2011/12

Blatt 1

(keine Abgabe)

Aufgabe 1 *(Should be avoided while driving...)*

a) A driver is driving "since forever" 20 m behind a truck (length 25m, 80 km/h). As he has 400 m of clear sight, he decides to overtake and accelerates with $a = 2m/s^2$ to 100 km/h. Is the overtaking successful?

b) As above, but with a tractor appearing at the end of the 400 m, having the speed 25 km/h.

c) That didn't work out, the car has to dodge and hits a tree at 100 km/h. Which height of free fall does the crash correspond to?

Aufgabe 2 *(Throwing as far as possible)*
A maiden stands on a 20 m high tower and wants to throw a ball, containing the key to her premises, as far as possible away from the castle. She is able to throw at 10 m/s. For which angle does the ball reach the furthest distance and what is this distance? (Hint: you are allowed to determine the maximum graphically.)

Aufgabe 3 *(Measuring errors)*

a) The two ends of a bridge of length L are seen an angle 2^o apart when watched from a point P 1 km away from the bridge's middle. To what accuracy can the length of the bridge be measured, if the angle measurement has a mean error of $1'$ (1 arc minute)?

b) The falling time of a sphere is being measured 40 times for a falling length of 1 m. The uncertainty in every measurement is 0.1 s. What is the accuracy of the mean falling time?

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 2

(Abgabe 02.11.2011)

Aufgabe 1 *(Fall off crane; 4 Punkte)*
A crane is lifting stones with constant velocity $v_z = 5m/s$ in vertical direction. When the stones reach a height of 6 m above ground, a stone loosens and falls off.

a) sketch graphically the height $z(t)$ of the stone, from the moment $t = 0$ on where the stone loosens. What is the maximum height of the stone?

b) At which time t_1 does the stone hit the ground and with which velocity?

Aufgabe 2 *(Airplane with side wind; 3 Punkte)*
A plane starts at airport A and has to reach airport B which lies 520 km away and exactly north of A. Neglect take off and landing and assume that the flight velocity (relative to the air !) is constantly 240 km/h. A wind from northwest is blowing with constant velocity 50 km/h.

a) Sketch the relevant velocity vectors.

b) Determine the course the plane has to head for to reach B.

c) Determine the time needed to reach B.

Aufgabe 3 *(Shooting exam; 3 Punkte)*
A recruit shoots with a canon at an angle of θ with the horizontal. The officer in the firing range aside (for simplicity: at the same point where the recruit is shooting) does not know the angle θ. Instead he measures the angle ϕ between the highest point of the trajectory of the canon ball and the horizontal.

a) Sketch the problem.

b) Show that the officer can control the proper adjustment of the canon by using the formula $\tan\theta = 2\tan\phi$.

Hint: Express ϕ by the maximum height h and the reach R of the trajectory. Then make a connection between h, R and θ. It holds $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 4

(Abgabe 16.11.2011)

Aufgabe 1 *(Loop; 3 Punkte)*
A mass point m has to perform a trajectory along a guide rail that consists of a horizontal rail followed by a circle (a loop; the plane of the circle is vertical, radius R).

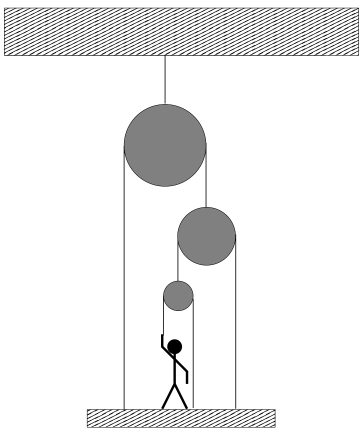
a) Determine the force normal to the guide rail that is pressing the mass point against the rail, as a function of angle ϕ in the circle ($\phi = 0$ at the lowest point of the circle). The horizontal initial velocity is v_0.

b) What is the minimal extension of a spring (spring constant D) to give the mass point an initial velocity so that it does not leave the circular guide rail?

c) At which point of the circular rail (at which angle ϕ_1) does the mass point fall off the rail when v_0 is chosen to be the velocity obtained by free falling the height $2R$?

(Remark: friction has to be neglected)

Aufgabe 2 *(Elevator; 2 Punkte)*
You ($M = 80$ kg) are standing in an elevator car of mass $m = 40$ kg, that is hanging via a system of rolls as sketched in the figure.



a) Which force is acting due to your own weight on the rope in your hand and which is the force that you exert on the car floor?

b) What is the maximum weight of the car that you can hold with your own weight? Why should small children not use this apparatus (physical reason, not pedagogical one)?

(Remark: for the gravitational acceleration you can use $g = 10\text{m/s}^2$)

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Aufgabe 3 *(Rocket; 5 Punkte)*
A rocket (mass $m_0 = 100$ t) is to be launched vertically. The velocity of the ejected gas relative to the rocket, $v_G = 4000$ m/s, is constant in time. Assume constant gravitational acceleration (independent of the height the rocket has reached) and neglect friction losses.

a) Which gas ejection rate $N_1 = -\frac{dm}{dt}$ is necessary for the rocket to just hover over the launch site (that means for the case that the velocity of the rocket is just $v_R = 0$)?

b) At what time $t_{1/2}$ does the rocket have left half of its initial mass, when the gas ejection rate is assumed to be constant and $N_2 = 500$ kg/s?

c) What is the acceleration of the rocket at $t_{1/2}$?

d) What is the speed of the rocket at $t_{1/2}$? Note that you have to use appropriate initial conditions.

e) What height above ground has the rocket reached at $t_{1/2}$?

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 5

(Abgabe 23.11.2011)

Aufgabe 1 *(Stationary Satellite; 4 Punkte)*
During the Apollo-Sojus mission in 1975, to connect to the headquarters a satellite over Africa was used that had a fixed position near the equator (so-called geostationary satellite, used for GPS nowadays).

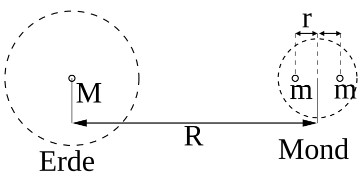
a) What is the distance r_s of the satellite from the earth's surface, its velocity v_s and which gravitational acceleration a_s is acting on it?

b) By means of steering noozles, the trajectory of the satellite is corrected. What are the changes in total energy, potential energy and kinetic energy, if the satellite (mass $m = 2$ t) is brought by $6,5$ km nearer to the earth?

c) How does this correction of trajectory affect the velocity and the orbital period?

Remark: the earth's radius is $r_E = 6,37 \cdot 10^6$ m.

Aufgabe 2 *(Tidal forces in the moon; 3 Punkte)*
Tidal forces describe differences in the gravitational forces in extended bodies. Their appearance can be illustrated by the following estimate:
Consider the earth as a point mass M. The moon is considered as divided into two halves and the two halves as two point masses m a distance $2r$ away from each other. The earth and the two 'halves of the moon' lie on a line and R is the distance between earth and moon (see sketch). F is the gravitational force between the two masses m and ΔF is the difference in the gravitational forces that the earth exerts on the two halves.



a) As an estimate, we assume that the two masses m have a stable configuration with respect to the tidal forces exerted by M, if $F \geq \Delta F$. Show that in that case the two masses must have a minimum distance R_0 from M.

b) Show that for $R_0 \gg r$ (R_0 much larger than r) it holds: $R_0 = \left(16\frac{M}{m}\right)^{1/3} r$.

c) According to this estimate, what distance does the moon ($M_M = 7,3 \cdot 10^{22}$ kg) has to have from the earth ($M_E = 6 \cdot 10^{24}$ kg) for it does not 'break' (radius of the moon $r_M = 1,74 \cdot 10^6$ m)? Compare to the actual distance.

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Aufgabe 3 *(Orbits and effective potential of the earth; 3 Punkte)*
Elliptical orbits are completely determined by their two semi-axes. Usually one writes $r_{max} = a(1+\epsilon)$ and $r_{min} = a(1-\epsilon)$ for the semi-major axis and semi-minor axis, where ϵ is called orbital eccentricity. For the earth one has $\epsilon = 0.0167$ and $r_{max} = 1AU$ (1 Astronomical Unit $= 1,496 \cdot 10^{11}$ m).

a) Determine the total energy E of the earth on its orbit around the sun as well as its angular momentum L, by using the effective potential and the above given data (to check and continue: one gets $E = -\frac{\gamma m M}{2a}$).

b) Assume that a cosmic villain wants to remove the earth from its stable orbit around the sun by leaving the angular momentum constant. How much additional energy is needed?

c) A second, energy-thirsty cosmic villain wants to extract energy from the earth's orbit and - again keeping the angular momentum constant - force it to a circular orbit. How much energy can he get out of it?

To get numbers, you need the following constants: mass of the earth $m = 5,97 \cdot 10^{24}$ kg, mass of the sun $M = 1,99 \cdot 10^{30}$ kg, gravitational constant $\gamma = 6,67 \cdot 10^{-11} \frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2}$.

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 6

(Abgabe 30.11.2011)

Aufgabe 1 *(Fictitious forces; 3 Punkte)*

a) You are sitting in a train that has the acceleration a. Describe how you can measure the acceleration by means of the magazine of the Deutsche Bahn and a pencil.

b) A train ($m = 2000$ t, constant velocity $v = 90$ km/h) is driving from North to South and crosses 60 degrees latitude. Determine the Coriolis force that the train exerts on the rails. Which mass does it correspond to?

c) Consider a cyclone at latitude $\phi = 67^o$. Near its borders, the wind may have a velocity of $v = 70$ km/h. Estimate the radius of the air masses that perform a circular motion in the horizontal plane.

Aufgabe 2 *(Tennis ball vs. basketball; 3 Punkte)*

A tennis ball (mass m) sits on top of a basketball (mass M much bigger than m; diameter d). The bottom of the basketball is a height h above ground, hence the bottom of the tennis ball has height $h + d$. The balls are dropped. Determine the height the tennis ball bounces, under the assumption that all collisions are elastic and that there is no friction. In doing so, transform to the center-of-mass system and use $M \gg m$. Remark: For simplicity, assume that the balls are separated by a very small distance, so that the relevant bounces (basket ball with ground and both balls against each other) happen a short time apart.

Aufgabe 3 *(Oblique impact; 4 Punkte)*

Two spheres of masses m_1 and m_2 collide in the x-y-plane elastically, but not centrally. In the laboratory frame, and before the collision, the first sphere has the velocity $\vec{u}_1$ in the x-y-plane, the second sphere is at rest, $\vec{u}_2 = 0$. The collision angle (with respect to the x-axis on which the centers of the spheres lie upon collision) is α.

a) Sketch the collision geometry. Which velocities $\vec{v}_1$, $\vec{v}_2$ and which angles β_1, β_2 with the x-axis (everything as a function of m_1, m_2 and α) do the spheres have after the collision? There is no friction at the spheres' surfaces. Hint: split the velocities into components perpendicular and parallel to the tangent plane of the spheres.

b) How big is the energy transfer during collision, compared to the initial energy? For which special case does the transfer reach its maximum value?

c) Discuss the case $m_2 \to \infty$ (second sphere = wall).

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Blatt 7

(Abgabe 07.12.2011)

Aufgabe 1 *(Rolling, rolling, rolling; 3 Punkte)*
A hollow cylinder having a thin wall (moment of inertia $I = mr^2$, mass $m = 20$ kg and radius $r = 40$ cm) is rolling without gliding on a horizontal surface. Take into account the friction force at the lowest point of the cylinder (where it touches the surface), which exerts a torque, $T = I\frac{d}{dt}\omega$ where ω is the angular velocity, and hence causes the rotation of the cylinder.

a) What is the acceleration a_s of the cylinder's center of mass, if a force $F = 40$ N is acting parallel to the ground and perpendicular to the cylinder axis (sketch!)? How big is the friction force F_r?

b) The maxmimum friction force is $F_r^{max} = \mu_0 mg$ with a friction coefficient $\mu_0 = 0,3$. What is the maximum force F for the cylinder not to start gliding?

c) Assume that the velocity of the center of mass is constant and $v_s = 10m/s$. Determine the total kinetic energy (rotation plus center of mass motion!), the momentum of the center of mass and the angular momentum of the cylinder.

Aufgabe 2 *(Center of mass of a cone; 3 Punkte)*
Consider a cone (more precisely a straight cone with circular base) of radius R and height $h = R$. The cone is homogeneous (meaning that its mass density is constant). Calculate the center of mass position $\vec{r}_s = (x_s, y_s, z_s)$. As the coordinate system, use the one where the base of the cone is in the x-y plane and where the apex is at $(0, 0, R)$.

Aufgabe 3 *(Principal moments of inertia of a cone; 4 Punkte)*
Consider the same cone as in the last problem. Determine the principal moments of inertia of the cone: i) the moment of inertia I_z around the z-axis (= axis through the apex and the center of the base) and ii) the moment of inertia $I_\perp$ around an (arbitrary) axis that is perpendicular to the z-axis and through the center of mass.

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Übungen zur Experimentalphysik I, WS 2011/12

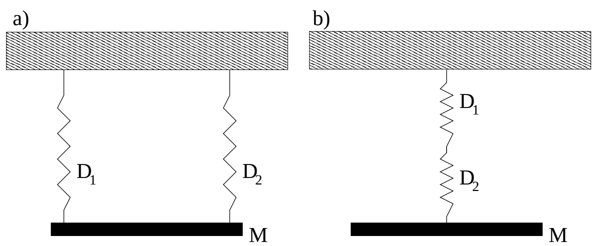
Blatt 8

(Abgabe 14.12.2011)

Aufgabe 1 *(Springs in parallel and in series; 4 Punkte)*
When modeling the elasticity of extended deformable objects, often models with several (or many) springs are used. Here we have a look at the simplest case (two springs).

a) Consider two springs (spring constants D_1 und D_2), that are in parallel, cf. Fig. a), and determine the effective spring constant of the combined system.

b) Consider two springs (spring constants D_1 und D_2), that are in series, cf. Fig. b), and determine the effective spring constant of the combined system.



Aufgabe 2 *(Spring with continuous mass; 2 Punkte)*

Calculate the total displacement of a spring (spring constant D), when the mass M is continuously distributed all along the spring. Compare the result with the (usually considered) case where the whole weight is hanging at the lower end of the spring.
Hint: think of the spring as being seperated in n small parts that are in series (cf. the problem above). Consider the total displacement and investigate the limit $n \to \infty$ (for a continuous mass distribution).

Aufgabe 3 *(Elongation of a rod; 4 Punkte)*

Consider a homogeneous rod made of steel (density $\rho = 7,8$ g/cm^3, length 10 m, elastic modulus $E = 200$ GPa, where 1 GPa$= 10^9$ Pa).

a) Calculate the elongation of the rod under its own weight for the case that the rod is hanging vertically (upper end is fixed).

b) The (assumed to be thin) rod is rotating with constant angular velocity $\omega = 0.5$ s^{-1} around an axis that is fixed to one end and perpendicular to the rod's axis. Calculate the tensile stress at any point inside the rod and the total elongation.

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Blatt 9

(Abgabe 21.12.2011)

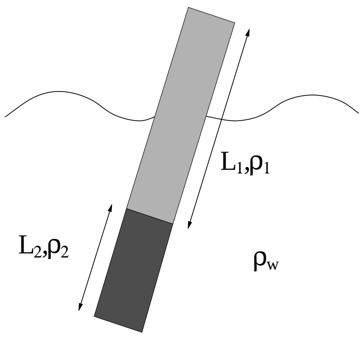
Aufgabe 1 *(Archimedes' principle; 3 Punkte)*

A cylindrical buoy is designed to float in a stable vertical position in water (density ρ_w). The upper part of the buoy has length L_1, cross sectional area A and constant density ρ_1. A second cylinder of length L_2, with the same cross section but with density ρ_2, is attached below, see sketch.

a) Assume that the buoy swims in vertical position. How long is its part outside of the water?

b) Assume the buoy is thin (rod-like, quasi-onedimensional). Calculate the center of mass of the vertically swimming buoy and the center of mass of the displaced water.

c) Sketch the slightly inclined buoy. Draw the force pair, acting on the two centers of mass (buoy and displaced water), and discuss the stability: what is the condition of stable vertical orientation?



Aufgabe 2 *(Surface tension; 4 Punkte)*

a) 1000 droplets of (liquid) mercury (radius $r_1 = 0,1$ mm, density $\rho = 13,55$ g/cm^3, surface tension $\sigma_0 = 0,46$ N/m) are merged into a single big drop, which can be regarded as a perfect sphere of radius R. How big is the released energy during this process? Compare the (excess) Laplace pressure in the small droplets with the one in the big drop.

b) Consider a free cylinder of liquid of radius R and length L. Free means that there are no walls, consider e.g. the water jet from a faucet. (i) Calculate the surface area A_z (neglecting top and bottom) of the cylinder. (ii) Calculate the surface $A(r)$ of n spherical drops of radius r, *that taken together have the same volume as the cylinder*. From the volume condition, you can eliminate n. (iii) A liquid jet from a faucet often is unstable and splits into drops. Discuss this observation by means of the surfaces calculated below: for which radius r^* does $A(r^*) = A_z$ hold?

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Aufgabe 3 *(Balloon; 3 Punkte)*

A spherical balloon with an opening at the bottom and an envelope (of fixed diameter 3m, mass of the envelope $m_H = 2$ kg) is filled with hydrogen. Assume throughout this problem that the temperature - independent of the height - is $T_0 = 0^o$ C.

a) What force is acting on the balloon when starting? Assume that the air pressure on the ground is $p_0 = 1$ bar (densities at p_0, T_0: air $\rho_L = 1,29$ kg/m^3; hydrogen $\rho_{H_2} = 0,09$ kg/m^3).

b) What height does the balloon reach? (Again, assume that $T = T_0$ and the gravitational acceleration are independent of height.)

Hint: Use Boyle's law and the barometric formula to get the height dependence of the densities.

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Blatt 10

(Abgabe 11.01.2011)

Aufgabe 1 *(Bernoulli in the fire hose; 4 Punkte)*

A fire fighter is holding a hose with internal cross section $A_S = 10\,\mathrm{cm}^2$ and a noozle with $A_D < A_S$. The rate of flow Q is 15 liters of water per minute. Above the noozle, there is a pressure drop of $\Delta p = p_S - p_D = 2$ bar.

a) With which velocities v_S and v_D is the water flowing in the horizontal hose and in the noozle? What is the diameter D of the noozle? (Sketch)

b) What is the rebound force the fire fighter experiences from the jet?

Aufgabe 2 *(Measuring the viscosity; 4 Punkte)*

There are many ways to measure the viscosity. Here are two simple ones:

a) One can show (G. Stokes, 1819-1903) that the friction force on a sphere in a laminar flow is given by $F_r = 6\pi\eta R v$, i.e. it is proportional to the viscosity η, to the radius of the sphere R and to the relative velocity v. A sphere of radius $R = 0,5$ mm made of lead (density $\rho_B = 11,3 \cdot 10^3 \mathrm{kg/m}^3$) is dropped in glycerin (density $\rho_G = 1,24 \cdot 10^3 \mathrm{kg/m}^3$). After short time a stationary velocity of $v = 6,5$ mm/s is measured. What is the viscosity of glycerin?

b) Consider the following viscosimeter: a circle-shaped disk of radius $R = 0,3$ m is rotating with angular velocity $\omega = 100 s^{-1}$ a distance of $d = 0,1$ mm above a plate that is parallel to the disk (sketch!). The gap between disk and plate is filled with an oil, whose viscosity is to be measured. To keep the rotation going, a torque of $T = 1,27 \cdot 10^4$ J is needed. What is the viscosity of the oil?

   Hint: as the distance between disk and plate is so small, the shear stress can be approximated as $\sigma = \eta \frac{v_{plate}}{d}$. Integrate the torque caused by this stress over the area of the disk.

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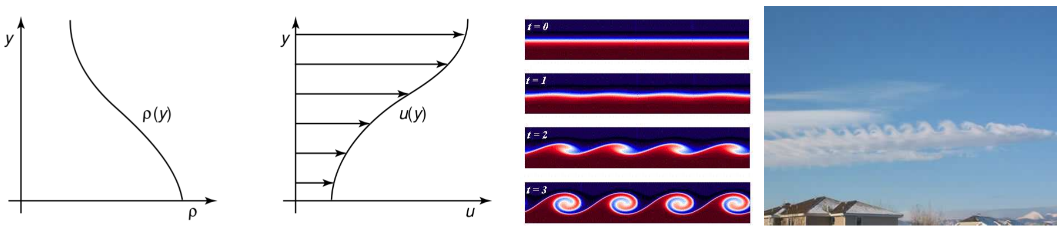

Abbildung 1: a) Density profile and velocity profile that together give rise to the KH instability. b) Numerical simulation (time evolution). c) The KH instability on the top of a cloud.

Aufgabe 3 *(Kelvin-Helmholtz instability; 5* **Zusatz***punkte)*

Consider a frictionless incompressible flow, where the density and the velocity have profiles as shown in the figure. This can occur, for instance, in clouds, on the surface of water (air on water) or in industriel processes of stratified (layer-like) systems. As is shown in the figures on the right, such profiles are often unstable and eddies form.
Here we want to understand this process by a very simplified reasoning. For this purpose we investigate the change in the kinetic energy and the buoyancy (everything is considered as per volume quantities), if we exchange two fluid volumes: one at y with density $\rho(y)$ and velocity $v(y) = v_1$; one at $y+\epsilon$ with $\rho(y+\epsilon)$ and $v(y+\epsilon) = v_1 + \Delta v = v_2$ (Sketch!).

a) Why does one have to consider the *exchange of two* volumes?

b) Does buoyancy act as a restoring force or is it causing the instability? Calculate the change ΔF of the buoyancy force under the assumption that ϵ is small [use the Taylor expansion $f(y) \simeq f(y_0) + \frac{df}{dy}(y - y_0)$]. Integrate over a small height distance Δy to find the performed work ΔW. Remark: the result is (for both volumes together!) $\Delta W = -g\frac{d\rho}{dy}(\Delta y)^2$.

c) Calculate the change in kinetic energy in two steps:

i) The energy before the volume exchange is $E_1 = \frac{\rho_0}{2}\left[v_1^2 + v_2^2\right]$. As the inhomogeneous density is mostly important for the buoyancy, here we assumed that the density is constant and $= \rho_0$.

ii) After the volume exchange we can assume that the fluid particles in the volumes acquired the averaged velocity, i.e. we write $E_2 = \frac{\rho_0}{2}\left[2\left(\frac{v_1+v_2}{2}\right)^2\right]$. Calculate $\Delta E = E_2 - E_1$ as a function of $\Delta v = v_2 - v_1$.

d) As a rough guess, the instability will set in when the change in kinetic energy related to the motion of the volumes is larger than the work of buoyancy (as the system then can perform mixing; why?). Show that this instability condition can be written as

$$-\frac{g}{\rho_0}\frac{d\rho}{dy}\frac{1}{\left(\frac{\Delta v}{\Delta y}\right)^2} < \frac{1}{4}$$

and discuss this result.

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 11

(Abgabe 18.01.2011)

Aufgabe 1 *(Damped harmonic oscillator; 6 Punkte)*

The harmonic oscillator is a very important model system, as oscillations occur in many systems (mechanical, electrodynamical, in molecules etc.). Although discussed already in the lecture, it will be investigated in more detail here.

a) The damped harmonic oscillator can be written as

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = 0\,. \tag{1}$$

By looking at the mechanical force balance, find expressions for β and ω_0 for the mass-spring system. Use the ansatz $x(t) \propto e^{\lambda t}$ to determine the two so-called eigenvalues λ_1 and λ_2 from Eq. (1). The general solution then reads $x(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t}$ (why?).

b) Consider the underdamped case ('Schwingfall'). In this case, λ_1 and λ_2 have imaginary parts, $\lambda_{1,2} = -\beta \pm i\omega$, with i the imaginary unit. Use $e^{iz} = \cos(z) + i\sin(z)$ to show that the general solution can be written as

$$x(t) = e^{-\beta t}\left(x_0 \cos\omega t + \frac{v_0 + \beta x_0}{\omega}\sin\omega t\right)\,, \tag{2}$$

where $x_0 = x(t = 0)$ and $v_0 = \dot{x}(t = 0)$.

c) A special case (critical damping, 'aperiodischer Grenzfall') is $\lambda_1 = \lambda_2 = \lambda$. For which relation between β and ω_0 does it occur? Calculate λ. In this case, the ansatz from above yields only *one* solution. Hence one uses the generalized ansatz $x(t) = a(t)e^{\lambda t}$. By insertion into Eq. (1), you get a simple equation for $a(t)$. Show that $a(t)$ is a linear function of t. The form of the oscillation strongly depends on the initial conditions x_0, v_0. Derive a condition for $x(t)$ passing through zero.

d) In the last, overdamped case ('Kriechfall') the damping is so large that both eigenvalues λ_1 and λ_2 are negative and real. Determine the general solution (again as a function of x_0, v_0) and the condition for $x(t)$ passing through zero.

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Aufgabe 2 *(Pendulum with additional mass; 2 Punkte)*

A homogeneous rod of constant cross section, length l and mass m is attached at one end so that it can swing and perform a harmonic oscillation. Now you are asked to add a mass $m_2 = am$ (a is an arbitrary numerical factor) to the pendulum so that the period of the oscillation is not changed. At which places, with respect to the swinging point, can you do this? Interpret the result.

Aufgabe 3 *(Superposition of oscillations; 2 Punkte)*

Consider the following superpositions of two harmonic oscillations:

a) Consider the resulting oscillation $x(t) = x_1(t) + x_2(t)$ from the superposition of two oscillations x_1 and x_2, that have the same period T, amplitudes $A_1 = 0.03$ m and $A_2 = 0.05$ m and a phase difference of $\phi = 60^o$. What is the amplitude and the phase angle of the superposition?

b) Consider the resulting oscillation, if x_1 and x_2 have same amplitudes and zero phase difference ($\phi = 0^o$). The frequencies f_1 and f_2 are almost equal. From the lecture we know that the superposition is a beat freqency ('Schwebung'). If you hear 5 beats per second and $f_1 = 40$ Hz, how big is f_2?

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Übungen zur Experimentalphysik I, WS 2011/12

Blatt 12

(Abgabe 25.01.2011)

Aufgabe 1 *(Driven pendulum; 3 Punkte)*

A mathematical pendulum of length $l = 1,2$ m is performing forced oscillations by harmonically displacing its attachment point A in horizontal direction: $x_A(t) = X_A \sin(\omega_A t)$ with amplitude $X_A = 3$ mm and period $T = 2$ s. Neglect friction effects.

a) Sketch the system and derive the equation of motion for the total displacement $x(t)$ of the pendulum (assume that all amplitudes are small).

b) What is the amplitude and the phase difference of this pendulum?

Aufgabe 2 *(Superposition of waves; 4 Punkte)*

a) Find $f_2(t,x)$ for a harmonic elastic wave such that, superposed with the wave $f_1(t,x) = A\cos(\omega t + kx + \phi_1)$, it results in the particle at position $x_0 = 1\,\mathrm{m}$ being stationary for all times [in other words, it results in a standing wave with a node ('Knoten') at x_0]. Consider $\omega = 10\pi\,\mathrm{s}^{-1}$, $k = \pi\,\mathrm{m}^{-1}$ and $\phi_1 = 60^o$.

b) A wave $f_E(t,x)$ of wavelength $\lambda = 0,28$ m is coming from far away and is moving in positive x-direction. At time $t = 0$ a wave maximum (anti-node, 'Wellenbauch') is passing $x = 0$ m. The wave is reflected from a fixed obstacle at $x_1 = 0,6$ m and hence a standing wave forms in the region $x < x_1$. Determine the positions of the nodes and maxima (anti-nodes), as well as the functional form $f(t,x) = f_E(t,x) + f_R(t,x)$ of the standing wave.

Aufgabe 3 *(Energy density of an elastic wave; 3 Punkte)*

A harmonic wave $f(t,x) = A\sin(kx - \omega t)$ is propagating inside an elastic medium (with modulus E and density ρ). Calculate the time-averaged (i.e. $\int_0^T ..\,dt$ with T the period) kinetic energy density, potential energy density and total energy density of the wave in the medium (energy density = energy/volume).
Hint for the potential energy: convince yourself that for an elastic medium, the analog of the displacement Δx of a spring is the quantity $\frac{\partial f}{\partial x}$ (why not simply f?). The elastic stress is then given by $\sigma = E\frac{\partial f}{\partial x}$.