Exercises for Advanced Particle Physics - Winter term 2013/14
Exercise sheet No. XI

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The solutions have to be returned to mail box no. 1
in the foyer of the Gustav-Mie-House before **Monday, February 3rd, 12:00h**.

Electroweak symmetry breaking

The gauge symmetry, on which the electroweak unification is based, is not compatible with short range interactions. In order to respect the gauge symmetry and to be in agreement with the observation of massive intermediate bosons, this symmetry has to be *spontanously* broken. A scalar field is then introduced along with its own dynamics to break $SU(2)_L \times U(1)_Y$: this is the Higgs mechanism.

1. Write down the Lagrangian of the electron field ψ interacting with the photon field A_μ. Show that this Lagrangian is invariant under the gauge transformation:

$$\left\{ \begin{array}{lll} \psi(x) & \rightarrow & \psi'(x) = e^{i\alpha(x)}\psi(x) \\ A_\mu(x) & \rightarrow & A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x) \end{array} \right. \qquad (1)$$

Show that an additional mass term for the photon would break this invariance. Is the gauge symmetry also broken by the mass term of the electron?

2. In the Standard Model, the electroweak bosons cannot be massive for the same reason. To have massive bosons and to keep the gauge symmetry, we introduce a doublet of scalar complex field Φ charged under $SU(2)_L \times U(1)_Y$ with the following Lagrangian:

$$\mathcal{L} \; = \; (D_\mu \Phi)^\dagger (D^\mu \Phi) \; - \; V(\Phi) \qquad (2)$$

where the following notations

$$\begin{aligned} \Phi &= \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \exp(\xi_i(x)\frac{\sigma^i}{2}) \begin{pmatrix} 0 \\ \rho(x) \end{pmatrix} && (3) \\ D_\mu &= \partial_\mu - ig'\frac{1}{2}B_\mu - igW^i_\mu \frac{\sigma_i}{2} && (4) \\ V(\Phi) &= \mu^2|\Phi|^2 + \lambda|\Phi|^4 && (5) \end{aligned}$$

The 2×2 matrices σ_i are the Pauli matrices, equation (3) represents a useful parametrisation of the four degree of freedom contained in Φ, (g', B_μ) and (g, W^i_μ) are the coupling constant and the gauge bosons associated to $U(1)_Y$ and $SU(2)_L$ respectively.

- By using equation (3), show that the field configuration which minimises V is

$$\Phi_0 = \begin{pmatrix} 0 \\ \sqrt{\frac{-\mu^2}{2\lambda}} \end{pmatrix} \equiv \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad (6)$$

- By developing the first term of equation (2) with $\Phi = \Phi_0$, show that a particular combination of gauge fields - to be defined - gets a mass.
  Hint: we introduce $W^\pm_\mu \equiv 1/\sqrt{2}\,(W^1_\mu \mp i W^2_\mu)$ and we recall that for charged particles, the mass term is $\mathcal{L}_m = m_W W^+_\mu W^{\mu -}$. In order to save time to diagonalise a matrix, we also give the following relation:

  $$(a,b)\begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = (A,B)\begin{pmatrix} \alpha_A & 0 \\ 0 & \alpha_B \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix}, \tag{7}$$

  where

  $$\begin{aligned} \alpha_A &= 0, & (8)\\ \alpha_B &= g_1^2 + g_2^2, & (9)\\ A &= \cos\theta\, a + \sin\theta\, b, & (10)\\ B &= -\sin\theta\, a + \cos\theta\, b, & (11)\\ \cos\theta &\equiv \frac{g_1}{\sqrt{g_1^2 + g_2^2}}. & (12) \end{aligned}$$

3. *Bonus.*
   Why is it not possible to include fermion masses before electroweak symmetry breaking?